Single-Constant Simplification of Kubelka-Munk Turbid-Media Theory for Paint Systems—A Review

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Abstract: For opaque coloration systems, Kubelka-Munk turbid media theory is used commonly to model optical mixing behavior. Most educational publications on the subject use opaque paint systems when describing the two-constant approach and textile systems when describing the single-constant simplification. Because of the differences in defining concentration for these systems and the corresponding degrees of freedom, the single-constant simplification for paint and textile systems are not identical. The second edition of "Principles of Color Technology" showed a numerical example for an opaque paint system modeled using the textile equations. The third edition used the same example but modified the degrees of freedom, a hybrid of the paint and textile approaches. Recent research by Berns and Mohammadi has evaluated the single-constant simplification for modeling artist paints; they have used both the hybrid and paint approaches. Thus, it was of interest to review these different approaches and determine whether these differences have practical importance and whether future printings and editions of Principles of Color Technology *should be modified. The three approaches were tested for tints made from a mixture of cobalt blue and titanium white acrylic emulsion artist paints. The differences between the textile and hybrid approaches were inconsequential. The paint approach was superior and its use is recommended for opaque paint systems. The differences in the numerical example from* Principles of Color Technology *were very small. For future printings of the third edition, the example will remain unchanged. For future editions, including the numerical example remains an open question.* © 2007 Wiley Periodicals, Inc. Col Res Appl, 32, 201–207, 2007; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/col.20309

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INTRODUCTION

Every spring quarter, the first-listed author teaches a laboratory-oriented course titled "Color Modeling" where different measurement and coloration systems are studied, both material and imaging, such as house paint, multichannel digital photography, and continuous and halftone printing. The models are both analytical and empirical and various statistical techniques are used including nonnegative least squares, piecewise linear regression, constrained nonlinear optimization, and principal component analysis. The course is based on a "generic approach to color modeling."[1]

The first laboratory is the "notorious paint lab," notorious because it is very time consuming, requires reasonable programming skills and familiarity with optimization, and can be plagued with experimental error because it is weight based and requires hand mixing of small amounts of material. Both the single- and two-constant forms of Kubelka-Munk (K-M) theory are used and the matching algorithms are spectral and colorimetric based. Although it is well known that the single-constant simplification of K-M theory is not appropriate for paint systems in the general case, this is the only practical way for the students to learn this approach and the use of piecewise linear regression.

Educationally, this has been a challenge because the single-constant simplification is shown in most textbooks only for textile systems. In this case, colorant concentration is a fraction of the amount of the textile substrate; the total degrees of freedom equal the number of colorants. Conversely, for opaque paints, colorant concentration is a fraction of the total amount of colorant; the total degrees of freedom equal the number of colorants minus one.

The second edition of *Principles of Color Technology*[2] introduced an example of determining the concentrations of

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an opaque paint composed of yellow, magenta, and white pigments, forming a brown color (pp. 160–161). Billmeyer and Saltzman assumed that the colored pigments contributed relatively small amounts of scattering and that the refractive index discontinuity between the paint film and air could be ignored. They used the textile system approach and determined the concentrations by solving two simultaneous equations. The third edition, *Billmeyer and Saltzman's Principles of Color Technology, 3rd Edition*,[3] used the same example and textile approach (pp. 166–167). Once the concentrations were determined, they were rescaled to reflect the reduced degrees of freedom. This was shown in the first and second printing. Several years ago, one of the modeling class students, Takayuki Hasegawa, pointed out that accounting for the reduced degrees of freedom needed to be carried through the entire example. The example was revised and this was shown in the third, and now, fourth printing. This will be referred to as the "hybrid" approach.

Very recently, we have been doing research to determine the minimum number of samples to characterize an artist paint's optical properties for both colorant selection for restorative inpainting (also known as retouching) and recipe prediction.[4,5] Single-constant K-M theory is appealing because the matching mathematics are much simpler. The first research[4] used the hybrid approach. When engaged in follow-on research,[5] Mohammadi was unable to derive the hybrid approach from the two-constant form of K-M theory. The derivation resulted in a slightly different equation, which was implemented in the follow-on research. There was a difference in scaling a pigment's mixture with white (i.e., a tint) when calculating a pigment's optical properties from two samples: a tint and white.

During the 1970s, Fred W. Billmeyer, Jr. and two of his graduate students from the Rensselaer Color Measurement Laboratory, Richard Abrams and Daniel Phillips, wrote a multipart set of articles about paint matching.[6–9] Part I[6] included the derivation of the single-constant simplification for opaque paints, corroborating the derivation. According to the authors, this approach was used in the COMIC I. This will be referred to as the "paint" approach.

Obviously, Billmeyer understood the difference between the textile and paint approaches when using the single-constant simplification since the second edition of *Principles of Color Technology* was published in 1981. Yet, the paint example used the textile approach. For this reason as well as the Billmeyer and Abrams article being somewhat obscure, it seemed worthwhile to review the K-M single-constant simplification for opaque paint systems. We were also interested in determining whether the differences between the textile, paint, and hybrid approaches were of practical significance.

THE SINGLE-CONSTANT SIMPLIFICATION

Consider a complex-subtractive color mixing system modeled using K-M theory. The additivity and linearity of the scattering and absorption coefficients of the individual constituents, k_λ and s_λ, respectively, to that of the mixture, $K_{\lambda,\text{mixture}}$ and $S_{\lambda,\text{mixture}}$ was expressed by Duncan[10] as

$$K_{\lambda,\text{mixture}} = \sum_i c_i k_{\lambda,i} \tag{1}$$

$$S_{\lambda,\text{mixture}} = \sum_i c_i s_{\lambda,i} \tag{2}$$

where c_i represents the concentration of each ith constituent. When we consider a mixture's ratio of the absorption and scattering coefficients, Eq. (3) results:

$$\left(\frac{K}{S}\right)_{\lambda,\text{mixture}} = \frac{\sum_i c_i k_{\lambda,i}}{\sum_i c_i s_{\lambda,i}} \tag{3}$$

The single-constant simplification assumes that the scattering results, predominantly, from one constituent and that the scattering from the other constituents is negligible. Equation (3) becomes

$$\left(\frac{K}{S}\right)_{\lambda,\text{mixture}} = \frac{c_1 k_{\lambda,1} + c_2 k_{\lambda,2} + \cdots + c_s k_{\lambda,s}}{c_s s_{\lambda,s}} \tag{4}$$

where the subscript s is used to denote the scattering constituent. Equation (4) is rewritten as

$$\left(\frac{K}{S}\right)_{\lambda,\text{mixture}} = \frac{c_1}{c_s}\left(\frac{k}{s_s}\right)_{\lambda,1} + \frac{c_2}{c_s}\left(\frac{k}{s_s}\right)_{\lambda,2} + \cdots + \left(\frac{k}{s_s}\right)_{\lambda,s} \tag{5}$$

It is the common practice to define a colorant's absorption and scattering properties as

$$\left(\frac{k}{s}\right)_{\lambda,i} = \left(\frac{k_i}{s_s}\right)_{\lambda} \tag{6}$$

This ratio, $(k/s)_\lambda$, is known as the colorant's "unit 'k' over 's'." The final equation for the single-constant simplification is

$$\left(\frac{K}{S}\right)_{\lambda,\text{mixture}} = \frac{c_1}{c_s}\left(\frac{k}{s}\right)_{\lambda,1} + \frac{c_2}{c_s}\left(\frac{k}{s}\right)_{\lambda,2} + \cdots + \left(\frac{k}{s}\right)_{\lambda,s} \tag{7}$$

THE TEXTILE APPROACH

For opaque textile, colorant concentration is expressed as a percentage of the total amount of the textile to be colored, often known as "owf" for on-weight-of-fabric:

$$c_i = \frac{(\text{weight})_i}{(\text{weight})_t} \tag{8}$$

where $(\text{weight})_i$ is the weight of an ith colorant and subscript t represents the substrate (i.e., fiber, yarn, fabric, etc.). By definition,

$$c_t = \frac{(\text{weight})_t}{(\text{weight})_t} = 1.0 \tag{9}$$

From Eqs. (8) and (9), we see that the number of degrees of freedom is equal to the number of colorants. For the textile approach, Eq. (7) becomes

$$\left(\frac{K}{S}\right)_{\lambda,\text{Textile}} = c_1\left(\frac{k}{s}\right)_{\lambda,1} + c_2\left(\frac{k}{s}\right)_{\lambda,2} + \cdots + \left(\frac{k}{s}\right)_{\lambda,t} \tag{10}$$

The scattering results from the substrate.

Under the assumption that the single-constant simplification models the textile's optical behavior, two samples are required to characterize a colorant's $(k/s)_\lambda$, the substrate ("blank dyed") and a single dyeing at an arbitrary concentration:

$$\left(\frac{k}{s}\right)_{\lambda,\text{Textile}} = \frac{(K/S)_{\lambda,\text{Dyeing}} - (k/s)_{\lambda,t}}{c} \quad (11)$$

where

$$\left(\frac{k}{s}\right)_{\lambda,t} = \left(\frac{K}{S}\right)_{\lambda,t} \quad (12)$$

THE PAINT APPROACH

For opaque paint, concentration is expressed as a ratio to the total amount of material:

$$c_i = \frac{(\text{weight})_i}{\sum_i (\text{weight})_i} \quad (13)$$

where $(\text{weight})_i$ is the weight of an ith constituent. As a ratio, the concentrations, by definition, sum to unity:

$$\sum_i c_i = 1.0 \quad (14)$$

From Eqs. (13) and (14), the number of degrees of freedom is one less than the number of constituents. For the paint approach, Eq. (7) becomes

$$\left(\frac{K}{S}\right)_{\lambda,\text{Paint}} = \frac{c_1}{c_w}\left(\frac{k}{s}\right)_{\lambda,1} + \frac{c_2}{c_w}\left(\frac{k}{s}\right)_{\lambda,2} + \cdots + \left(\frac{k}{s}\right)_{\lambda,w} \quad (15)$$

where subscript "w" represents a white, highly scattering constituent.

Under the assumption that the single-constant simplification models a paint's optical behavior, two samples are required to characterize a colorant's $(k/s)_\lambda$, a white and a single mixture at an arbitrary concentration, known as a tint:

$$\left(\frac{k}{s}\right)_{\lambda,\text{Paint}} = \frac{(K/S)_{\lambda,\text{tint}} - (k/s)_{\lambda,w}}{c_{\text{tint}}/c_w} \quad (16)$$

where

$$\left(\frac{k}{s}\right)_{\lambda,w} = \left(\frac{K}{S}\right)_{\lambda,w} \quad (17)$$

This derivation was described in the Billmeyer and Abrams article[6] and used in the Mohammadi and Berns research.[5]

THE HYBRID APPROACH

This approach begins with the textile mixing equation, Eq. (10), and accounts for the reduced degrees of freedom, in which the total concentration is unity [Eq. (14)]:

$$\left(\frac{K}{S}\right)_{\lambda,\text{Hybrid}} = c_1\left(\frac{k}{s}\right)_{\lambda,1} + c_2\left(\frac{k}{s}\right)_{\lambda,2} + \cdots + (1 - c_1 - c_2 - \cdots)\left(\frac{k}{s}\right)_{\lambda,w} \quad (18)$$

Thus, the equation to calculate a colorant's $(k/s)_\lambda$ is

$$\left(\frac{k}{S}\right)_{\lambda,\text{Hybrid}} = \frac{(K/S)_{\lambda,\text{tint}} - (1 - c_{\text{tint}})(k/s)_{\lambda,w}}{c_{\text{tint}}} \quad (19)$$

This approach was used in the third and fourth printing of *Billmeyer and Saltzman's Principles of Color Technology, 3rd edition*[3] and research by Mohammadi and Berns[4] and Okumura.[11]

TESTING THE THREE APPROACHES FOR PAINT

The equations for determining a colorant's $(k/s)_\lambda$, using the textile, paint, and hybrid approaches are rewritten in Eqs. (20)–(22), respectively, to better reveal the differences:

$$\left(\frac{k}{s}\right)_{\lambda,\text{Textile}} = \frac{1}{c_{\text{tint}}}\left(\frac{K}{S}\right)_{\lambda,\text{tint}} - \frac{1}{c_{\text{tint}}}\left(\frac{k}{s}\right)_{\lambda,w} \quad (20)$$

$$\left(\frac{k}{s}\right)_{\lambda,\text{Paint}} = \frac{1-c_{\text{tint}}}{c_{\text{tint}}}\left(\frac{K}{S}\right)_{\lambda,\text{tint}} - \frac{1-c_{\text{tint}}}{c_{\text{tint}}}\left(\frac{k}{s}\right)_{\lambda,w} \quad (21)$$

$$\left(\frac{k}{s}\right)_{\lambda,\text{Hybrid}} = \frac{1}{c_{\text{tint}}}\left(\frac{K}{S}\right)_{\lambda,\text{tint}} - \frac{1-c_{\text{tint}}}{c_{\text{tint}}}\left(\frac{k}{s}\right)_{\lambda,w} \quad (22)$$

Comparing textile and paint, there are different scaling functions, $\frac{1}{c_{\text{tint}}}$ vs. $\frac{1-c_{\text{tint}}}{c_{\text{tint}}}$. At very low concentrations, the two functions are nearly coincident. When the tint concentration is unity, the textile function is unity while the paint function is zero. At low tint concentrations, the difference is marginal. At high tint concentrations, the difference is appreciable. These two functions are plotted in Figure 1 for concentrations ranging between 0.01 and 0.99. Plotting the ordinate on a logarithmic scale reveals the divergent behavior at high concentrations. The hybrid approach scales the tint using the textile function and scales the white using the paint function.

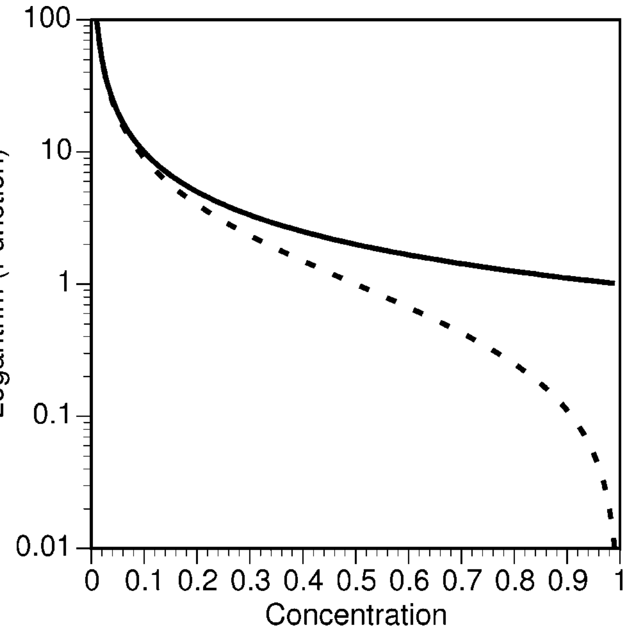

FIG. 1. Relationship between the chromatic concentration in a tint, c_{tint}, and $\frac{1}{c_{\text{tint}}}$ (solid line) or $\frac{1-c_{\text{tint}}}{c_{\text{tint}}}$ (dashed line).

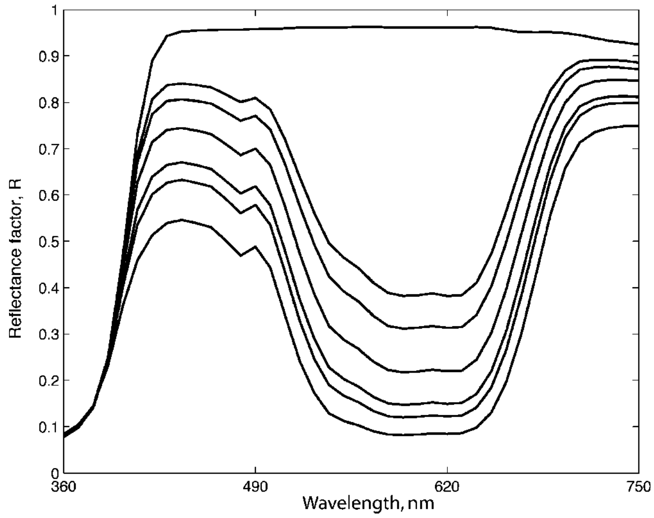

FIG. 2. Spectral reflectance factor of Liquitex Acrylic Artist Color cobalt blue, titanium white, and various tints of the two.

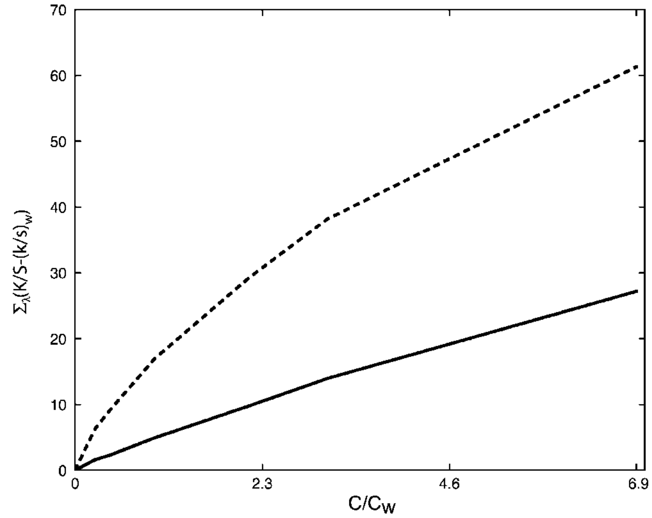

FIG. 4. Optical behavior of cobalt blue tints based on the paint approach both including (solid line) and omitting (dashed line) the Saunderson correction.

Research was performed to evaluate the accuracy of K-M theory when modeling the optical behavior of artist acrylic emulsion paints[4,11] and the effects of varnishing on color and appearance.[11] Drawdowns were prepared using Liquitex Acrylic Artist Color Professional High Viscosity paint. For this publication, only cobalt blue is evaluated. The drawdowns were opaque and without surface imperfections. Concentration was defined by weight. Samples were measured using a GretagMacbeth Color Eye XTH integrating sphere spectrophotometer with specular component included. The spectral reflectance factor measurements for cobalt blue (masstone), titanium white (masstone), and six tints at different concentrations are plotted in Fig. 2.

The conversion from reflectance factor, R_λ, to $(K/S)_\lambda$ values often includes accounting for the refractive index discontinuity between the paint medium and air at the paint surface. The Saunderson correction[12] was used to calculate internal reflectance factor with K_1 and K_2 defined as 0.03 and 0.65, respectively

$$R_{\lambda,i} = \frac{R_{\lambda,m} - K_1}{1 - K_1 - K_2 + K_2 R_{\lambda,m}} \tag{23}$$

where the subscripts m and i refer to "measured" and "internal," respectively. (As referenced by Saunderson, this equation was derived and first published by Ryde.[13]) The Saunderson constants were optimized to maximize modeling accuracy for the two-constant form of K-M theory for this type of paint system.[11] $(K/S)_\lambda$ was calculated in the usual manner:

$$\left(\frac{K}{S}\right)_\lambda = \frac{(1 - R_{\lambda,i})^2}{2R_{\lambda,i}} \tag{24}$$

The appropriateness of the single-constant simplification may be evaluated by plotting the numerator vs. the denominator of the right-hand side of Eqs. (11), (16), or (19) for a

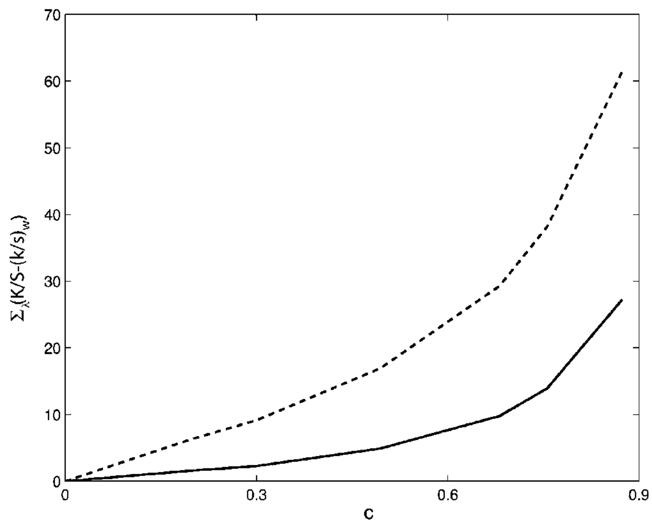

FIG. 3. Optical behavior of cobalt blue tints based on the textile approach both including (solid line) and omitting (dashed line) the Saunderson correction.

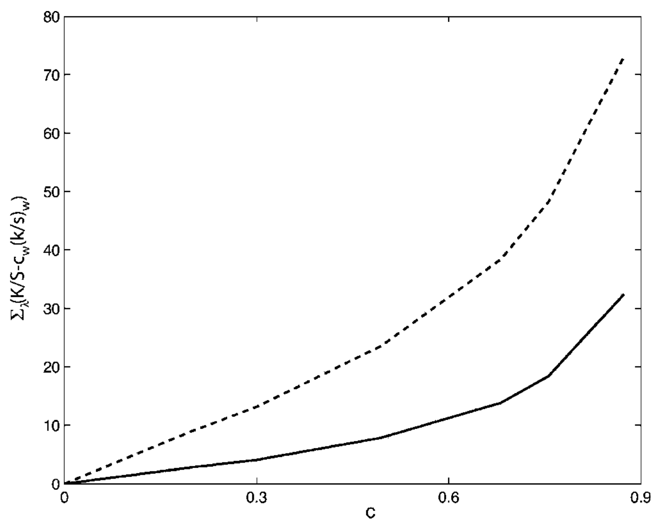

FIG. 5. Optical behavior of cobalt blue tints based on the hybrid approach both including (solid line) and omitting (dashed line) the Saunderson correction.

TABLE I. Spectral %RMS error statistics between measured and predicted spectral reflectance factor using $(k/s)_\lambda$ of each of the listed tints to predict the remaining tints using actual concentrations.

	All tints	$L^* = 76$, $C^*_{ab} = 28$	$L^* = 72$, $C^*_{ab} = 32$	$L^* = 65$, $C^*_{ab} = 40$	$L^* = 58$, $C^*_{ab} = 45$	$L^* = 54$, $C^*_{ab} = 48$	$L^* = 47$, $C^*_{ab} = 51$	Grand average
Textile model								
Average	5.9184	5.5842	5.1092	4.3697	4.4202	5.2176	8.6050	5.60
Maximum	10.5335	13.6881	13.0421	10.7734	7.1575	9.4913	14.5812	11.32
Std. dev.	3.7698	5.2446	4.9981	3.6657	2.9513	3.6610	5.5549	4.26
Hybrid model								
Average	5.9184	5.5842	5.1092	4.3697	4.4202	5.2176	8.5900	5.60
Maximum	10.5335	13.6881	13.0421	10.7734	7.1575	9.4913	14.5813	11.32
Std. dev.	3.7698	5.2446	4.9981	3.6657	2.9513	3.6610	5.5832	4.27
Paint model								
Average	1.9239	2.7575	2.0776	1.4940	1.3106	1.3668	2.1955	1.88
Maximum	3.1220	6.3774	5.1962	3.7206	2.4046	2.3588	3.4569	3.81
Std. dev.	1.0434	2.3895	1.9387	1.2601	0.8965	0.8812	1.2671	1.38

wavelength where absorption is maximized. However, the maxima often shift when using the single-constant simplification because the model assumptions often are not met. This is remedied, typically, by integrating the spectra as a function of wavelength, often referred to as "sum 'k' over 's'." This is shown in Figures 3–5. The cobalt blue masstone is excluded from these plots since the single-constant simplification assumes that the scattering is a result of titanium white, obviously absent from the masstone. These calculations were also performed without accounting for the refractive index discontinuity; that is, it was assumed that internal and measured reflectance factor were identical. The first observation is that linearity was improved with the use of the Saunderson correction. This correction is almost always used when the measurement geometry is integrating sphere with the specular component included, precisely for this reason.

The second observation is that for the paint approach, the curves were compressive whereas for the textile and hybrid approaches, the curves were expansive. We would expect that with increasing concentration, a colorant's ability to absorb light decreases, resulting in a compressive curve. This is typical behavior for textile dyes using the single-constant simplification and pigments used in coatings using the two-constant approach. It was also observed that the paint approach resulted in the best linearity. This simple graphical evaluation revealed that the paint and textile approaches were quite different, because of the different scaling: c/c_w vs. c. The textile and hybrid approaches were very similar because the abscissas had the same scale. Differences in the ordinates had a small impact on linearity because the $(k/s)_{\lambda,w}$ was quite small.

A quantitative test was performed where each tint was used, successively, to calculate $(k/s)_\lambda$. In addition, all of the tints were used to calculate $(k/s)_\lambda$ using nonnegative least squares with the objective function minimizing (K/S) mean-square error for the entire tint ladder. In all cases, the Saunderson correction was used. This was repeated for each approach, resulting in $21(k/s)_\lambda$ determinations. Since the recipe of each tint was known, spectral reflectance factor was calculated for each tint using the appropriate equations. The actual and calculated spectra were analyzed for similarity using spectral reflectance factor %RMS error. The statistical results are shown in Table I. The textile and hybrid approaches were nearly identical (hence the many significant figures listed). The slight difference in the amount of white subtracted from the scaled tint when calculating $(k/s)_\lambda$ was inconsequential. The paint approach exhibited the best performance. In similar fashion to our other research,[5] there is a trend where the performance is optimal in the region where chroma maximizes.

TABLE II. Spectral %RMS error statistics between measured and predicted spectral reflectance factor using $(k/s)_\lambda$ of each of the listed tints to predict the remaining tints using effective concentrations.

	All tints	$L^* = 76$, $C^*_{ab} = 28$	$L^* = 72$, $C^*_{ab} = 32$	$L^* = 65$, $C^*_{ab} = 40$	$L^* = 58$, $C^*_{ab} = 45$	$L^* = 54$, $C^*_{ab} = 48$	$L^* = 47$, $C^*_{ab} = 51$	Grand average
Textile model								
Average	1.1539	3.7845	3.3659	2.3590	1.4532	1.2947	0.9864	2.06
Maximum	2.6206	12.3016	11.6636	9.4073	5.7601	3.7010	1.6124	6.72
Std. dev.	0.8621	4.7431	4.5307	3.6175	2.1402	1.2794	0.6300	2.54
Hybrid model								
Average	1.1539	3.7845	3.3659	2.3590	1.4532	1.2947	0.9723	2.05
Maximum	2.6203	12.3016	11.6636	9.4073	5.7601	3.7010	1.6127	6.72
Std. dev.	0.8618	4.7431	4.5307	3.6175	2.1402	1.2794	0.6561	2.55
Paint model								
Average	0.8739	1.2995	0.9975	0.7196	0.6466	0.7464	0.9864	0.90
Maximum	1.4906	2.6267	2.1451	1.4956	1.0238	1.3326	1.6124	1.68
Std. dev.	0.5743	1.0443	0.8533	0.5189	0.3827	0.5235	0.6300	0.65

TABLE III. Correlation (r^2) and slope statistics of linear regression fits between actual (independent variable) and effective (dependent variable) concentrations.

	Textile model		Hybrid model		Paint model	
	r^2	Slope	r^2	Slope	r^2	Slope
Average	0.8997	1.23	0.8997	1.23	0.9994	0.92
Minimum	0.8646	1.10	0.8646	1.10	0.9978	0.91
Maximum	0.9499	1.33	0.9499	1.33	0.9999	0.93
Std. dev.	0.0359	0.09	0.0359	0.09	0.0008	0.01

A second quantitative test was performed where nonlinear optimization was used to estimate the effective concentration for each tint using the $(k/s)_\lambda$ and the appropriate mixing equation for each approach. The objective function was minimizing sum-of-square spectral reflectance factor error. The term "effective" refers to the optical effect of a colorant within the mixing system.[1] Quite often a coloration system has a nonlinear relationship between the scalars of a spectral model and the user controls of the coloration process.[1] The effective concentrations are a determination of these scalars. Their use rather than the actual concentrations compensate for experimental errors and the nonlinear relationship. The results are shown in Table II. Performance is improved for all three approaches, as expected. The trends described using the actual concentrations (Table I) occurred also for the effective concentrations.

If we assume that any experimental errors are small and random, the relationship between actual and effective concentrations are a measure of how well these two paints' mixing behavior is modeled using the single-constant simplification of K-M theory. For each set of effective concentrations, the linear relationship was evaluated by fitting a straight line with zero offset using linear regression with the actual concentration the independent variable and the effective concentration the dependent variable. The slope and correlation coefficient, r^2, were used as performance measures. The aim results were both a slope and r^2 of unity. The masstone was included in this analysis. The results are given in Table III. For the textile and hybrid approaches, the r^2 was 0.8997 with a slope of 1.23, on average. The slope above unity represents the expansive relationship, for example, as plotted in Figures 3 and 5. For the paint approach, the r^2 was 0.9994 and a slope of 0.92, on average. This indicates a slight compressive behavior and that the single-constant simplification had reasonable performance.

CONCLUSIONS

Three approaches to the single-constant simplification of K-M theory were described in this article. The textile and paint approaches were derived from the two-constant equations. The final equations for textile are shown in most texts concerned with this subject while the paint approach is rarely shown. A third approach, the hybrid approach, was a modification of the textile approach with constrained concentrations as used in the paint approach. All three approaches have been used to model opaque paint mixing systems.

The three methods were tested using cobalt blue and titanium white acrylic emulsion artist paints. Six tints were prepared at various ratios of blue to white. Model effectiveness was evaluated using graphical and quantitative methods. The differences between the hybrid and textile approaches were negligible. Both methods were inferior to the paint approach. This has also been validated for other pigments.[5] When using the single-constant simplification for opaque paints, the paint approach is recommended.

One of our objectives when undertaking this analysis was to determine if differences mathematically would lead to practical differences. Billmeyer and Saltzman used the textile approach[2]; perhaps they knew that the difference was insignificant. As we showed, the paint approach was clearly superior to the textile approach. We believe they used the textile approach because it is simpler. One of their philosophies when writing their book as well as guiding the writing of the third edition was the "KISS" principle, "keep it simple, stupid." The concepts being given in the numerical example are the same regardless of approach.

This still leaves open the question of future printings and editions of *Billmeyer and Saltzman's Principles of Color Technology*. For future printings, the hybrid approach will remain in place. As shown in the Appendix, the differences in all three methods for the numerical example are trivial. For the fourth edition, the path is less clear. The first edition did not contain any numerical examples. Given today's computer processing and the unlikelihood of anyone solving two simultaneous equations by hand, perhaps these numerical examples are unnecessary. (There is also a hand calculation for Beer's law.) The first listed author of this publication would be interested in any opinions.

APPENDIX

The numerical example from *Principles of Color Technology* was repeated using each of the three approaches. The

TABLE AI. Spectral data for numerical example from *Principles of Color Technology.*[2,3]

	Reflectance factor		(K/S)	
	420 nm	560 nm	420 nm	560 nm
Y	0.216	0.872	1.423	0.009
M	0.384	0.146	0.494	2.498
B	0.167	0.163	2.078	2.149
W	0.768	0.882	0.035	0.007

TABLE AII. Concentration (percentage) results for brown mixture using the textile, paint, and hybrid approaches.

Approach	C_y	C_m
Textile	21.8	11.7
Paint	21.2	11.9
Hybrid	21.8	11.7

spectral data are given in Table AI. "Sample W contains white pigment only. Sample Y contains 18.5% yellow pigment in white, while sample M contains 13.6% magenta pigment in white. Sample B, brown in color, contains unknown percentages of the yellow, magenta, and white pigments."[2,3] To further quote the book, "we leave it to the reader to confirm …," in this case, the results given in Table AII. In our case, Matlab was used.

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